Related rates look at the relationships between rates of change (or rates of derivatives). This exercise section demonstrates finding related rates with three practice problems.

# Air Filling a Balloon

Figure

Air is pumped into a balloon at a rate of . How fast is the radius of the balloon increasing when the diameter is ? See Figure 1.

Known information:

|  |  |
| --- | --- |
| English | Math |
| Rate of increase of volume () of air is . |  |

Unknown information:

|  |  |
| --- | --- |
| English | Math |
| Rate of increase of radius () when the diameter is . |  |

If the balloon is a sphere, , then the derivative of volume over radius is

Solving for the unknown gives:

# Ladder Sliding from a Wall

A ladder leans on a wall. It stands 6 feet from the wall, and it is 10 feet long. The ladder is slipping away from the wall at the speed of 1 foot per second. How fast is the top of the ladder sliding down the wall? (See **Error! Reference source not found.**)

To start, let’s make a Pythagorean triangle of the wall, floor, and ladder. See Figure 2.

Figure

Figure

Known information:

|  |  |
| --- | --- |
| English | Math |
| Rate of sliding on floor () is . |  |

Unknown information:

|  |  |
| --- | --- |
| English | Math |
| Rate of sliding on wall () when the foot of the ladder is from the wall (). |  |

Since , , and form a Pythagorean triangle, . Therefore, when , . The derivative of , , can be found using implicit differentiation:

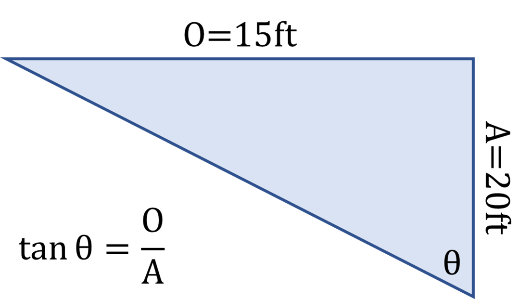
Back to the original problem,

Taking the already-calculated ,

# Swiveling Spotlight

A man walks on a line at the speed of . A spotlight shines on the man from a certain point on the path (from the spotlight to the point is perpendicular to the path). When the man is from the point, how fast will the spotlight be rotating? See Figure 4.

Figure

This problem can be solved with trigonometry. Constructing a triangle from Figure 4, an angle tells the sweep from the spotlight to the man and the point the spotlight is based off. See Figure 5.

Figure

Known information:

|  |  |
| --- | --- |
| English | Math |
| Rate of motion away from light () is |  |

Unknown information:

|  |  |
| --- | --- |
| English | Math |
| Rate of swiveling () when the man of the ladder is () from the point on the line that the light is from (). |  |

Differentiate both sides over time () with the chain rule.

Rearrange terms algebraically.